**CMPT 335 Discrete Structures**

**Final Exam**

There are 10 problems. The total cost of all of them is **130 points**.

Solving any 8 of these problems you will get not less than 95 points and solving any 9 of them you will get over 100 points. Good luck!

**Please, submit your solutions in a single MS Word file named FirstName\_LastName\_Final**.

**Any scanned pictures shall be included in that file and MUST be clearly readable**.

1. (**10 points**) Prove using mathematical induction that

Basis: S0= = = 1

Hypothesis: Sn = Sn+1 =

Sn→ Sn+1 Sn+1 = Sn + qn+1 = + qn+1 = =

**2**.(**10 points**) Prove using mathematical induction that

Basis:

If n = 1, then 2=2^(2)-1 = 3

Hypothesis: Assume

Step(show P(n)→P(n+1) is true)

**3**. (**10 points**) Prove the following generalization of the De Morgan’s law using mathematical induction



P(n): = ∧… ∧

Basis: =

Hypothesis: = = … = …

Step:

= = = =

… = …

**4**. (2**0 points**) Verify the correctness of the Selection Sort algorithm (**descending** order) using induction.

Algorithm:

**for** j = 1 to **length** (A)-1

{

**for** i=j+1 to **length (**A)

**if** A[j]<A[i] // if this is true, swap A[i] and A[j]

{

key=A[j]

A[j]=A[i]

A[i]=key

}

}

}

Basis: Suppose an array consists of 2 elements. The key is A[j]. If A[j] is less than A[i], swap A[i] and A[j], thus making the key A[i]. If A[j] is greater than A[i] don’t swap A[i] and A[j].

Hypothesis: Suppose this algorithm may sort an array containing n elements.

Step: We have an array containing n+1 elements. Suppose first n of them are already sorted (based on the inductive step). Let us consider the last iteration of the for loop, j=n-1. Then i=n.

The key is A[n-1]. If A[n-1] is less than A[n], swap A[n] and A[n-1], thus making the key A[n]. Since first n elements are already sorted, this means that an entire array is already sorted. If A[n-1] is greater than A[n], don’t swap them.

**5**. (**10 points**) Evaluate the following (justify your solution)

1.  in 

[4] + [19] = [23] = [3] in because 23mod5=3

1.  in 

= [76] = [1] in  because 76mod5=1

1.  in 

= [170] = [2] in  because 170mod3=2

1. in 

≡ ≡ [1] ≡ [1] in 

1. in 

≡ ≡ [1] ≡ [1] in 

**6.** (**15 points**) A message was encrypted using a simple encryption method with the encryption function (key) . The following encrypted message was obtained HNRTRNKC Invert the encryption function (key) and decrypt the message. Show your work explicitly (including a decrypted message).

Ciphertext: HNRTRNKC

Decryption using p=3(r-13)mod26

H

r=7

p=3(r-13)mod26

p=3(7-13)mod26

p=-18mod26

p=8

I

N

r=13

p=3(r-13)mod26

p=3(13-13)mod26

p=0mod26

p=0

A

R

r=17

p=3(r-13)mod26

p=3(17-13)mod26

p=12mod26

p=12

M

T

r=19

p=3(r-13)mod26

p=3(19-13)mod26

p=18mod26

p=18

S

K

r=10

p=3(r-13)mod26

p=3(10-13)mod26

p=-9mod26

p=17

R

C

r=2

p=3(r-13)mod26

p=3(2-13)mod26

p=-33mod26

p=19

T

Plaintext: IAMSMART

**7**. (**15 points**) Draw a simple graph  where 

**8**. (**15 points**) Prove isomorphism of the following two simple graphs (verify isomorphism invariants and show 1-to-1 correspondence between vertices explicitly (like A2 = f(A1) )



They are isomorphic because they have a one to one correspondence(bijection) between their vertices which is shown below.

f(A4)

f(A2)

f(A1)

A1

A2



A3

A6

f(A6)

f(A5)

f(A3)

A5

A4

**9**. (**15 points**) Draw a pseudo graph with the set *V* of vertices, the set *E* of edges, find its adjacency matrix, find degrees of all its vertices and verify the Handshaking Theorem and the Theorem about an adjacency matrix.



0 1 0 1 2 0

1 0 1 0 0 0

0 1 0 2 0 0

1 0 2 0 1 2

2 0 0 1 2 0

0 0 0 2 0 2

deg(A) = 4

deg(B) = 2

deg(C) = 3

deg(D) = 6

deg(E) = 5

deg(F) = 4

Ag =

The Handshaking Theorem:

∑ deg(V)=2E

24 = 2(12)

24 = 24 ✓

Theorem about an adjacency matrix:

The sum of the entries in row i of the adjacency matrix of a simple graph is the degree of the i^th vertex.

0 1 0 1 2 0

1 0 1 0 0 0

0 1 0 2 0 0

1 0 2 0 1 2

2 0 0 1 2 0

0 0 0 2 0 2

deg(A) = 4; deg(B) = 2; deg(C) = 3; deg(D) = 6; deg(E) = 5; deg(F) = 4

Ag =

**10**. (**10 points**)

a) Draw a tree matching the following parenthetical notation 

b) Create a parenthetical notation for the following tree

C

A

B

D

F

E

G

J

H

K

**A(B(CD(HK))E(FGJ))**